Fee versus royalty licensing in a Cournot competition model:

An extension to the case of J firms

1. Introduction

When looking at situations involving the Cournot duopoly, we generally structure the cost function so that the firm produces at a constant marginal cost. However, if a firm is able to reduce their unit cost, either through being an innovator or through purchasing that innovation, the dynamics of the market can significantly change. This can potentially provide an advantage to those firms willing to acquire cost reducing innovations. As the real world instances of pure duopoly are rare, it is often beneficial to extend a situation presented in a Cournot duopoly model to the case of “J” firms in order to see how the described behavior withstands the adjustment to increased competition.

This paper will extend the model presented in *Fee versus royalty licensing in a Cournot duopoly model* by X. Henry Wang from a duopoly to the case of an arbitrary number of firms. By doing so I will validate his argument that for an innovator who is also a producer within an industry, licensing an innovation to the competition by means of a royalty is superior to licensing by a fixed fee. I will also show that this result holds even as the number of firms in the industry approaches infinity, as result which contradicts the assertion of Wang.

Similar to the model put forth by Wang, this exercise considers the case of “J” firms producing a single homogeneous good, where one firm within the industry has developed a cost reducing innovation. The main difference between Wang’s model and the existing literature is the inclusion of the innovating firm as an industry producer, rather than simply an outside firm licensing the innovation to the industry. This change is what accounts for the superiority of royalty licensing over fixed fee licensing.
2. Brief history concerning the model

The Cournot model of competition has proved itself to be very adaptable in the pursuit of explaining market behavior. Cournot’s original model portrayed a duopoly in which firms non-cooperatively competed on the basis of quantity, in one homogenous good with identical constant marginal costs. Though on the surface its reliance on firms choosing their quantity, rather than their price, seems less realistic, the equilibrium outcomes portray reality better than the comparable Bertrand. In the Bertrand model, as soon as two firms exist in a market competing on price, the only equilibrium is when they both produce at their marginal cost. The realism that is given up by competing in terms of quantity is acceptable, given the better explanatory power of the Cournot model’s equilibrium outcomes.

3. Presentation and evolution of the model

We start by constructing the typical case of a Cournot competition model with J firms. The inverse demand function for the market is \( p = a - Q \) where \( p \) is price and \( Q \) is the output of all firms in the market. Before any innovation, all firms produce at a constant marginal cost per unit of \( c \), where \( 0 < c < a \). The innovation enters as a reduction in the unit cost, denoted by \( \epsilon \). The innovator, firm 1, sets the price of the royalty or fee to maximize total income.

To establish a point of comparison, and the basic model from which we will elaborate, consider the normal case of Cournot competition where each firm has a constant unit cost, denoted \( c_j \). The profit function for firm 1 is \( \pi_1 = (a - q_1 - \sum_{j=2}^{J} q_j - c_1) q_1 \). Taking the first order conditions of the profit function with respect to \( q_1 \) yields firm 1’s reaction function \( a - 2q_1 - \sum_{j=2}^{J} q_j - c_1 = 0 \), which can be rewritten as \( a - q_1 - Q = c_1 \). Summing across the first order conditions for all firms gives us \( J a - Q - \int Q = \sum_{j=1}^{J} c_j \). Solving this equation for the market
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quantity Q gives the result \( Q = \frac{ja-\sum_{j=1}^{J} c_j}{(J+1)} \) and the market price, \( P = \frac{a+\sum_{j=1}^{J} c_j}{(J+1)} \). Using these expressions we can find the equilibrium quantity and profit for any firm in the industry as:

\[
q^*_j = \frac{a-Jc_j-c_j+\sum_{j=1}^{J} c_j}{(J+1)}, \quad \text{and} \quad \pi^*_j = \left[ \frac{a+\sum_{j=1}^{J} c_j-c_j(J+1)}{(J+1)} \right]^2.
\]

Having established the basic model, we now turn to the case where firm 1 has developed a cost reducing innovation. The innovation must be minor \( \varepsilon < a - c \) or it will be considered a disruptive innovation which will result in firm 1 becoming a monopoly, an idea beyond the scope of this exercise. We will first consider when firm 1 implements the innovation in its own production, but either chooses not to license it to any other firm or the other firms decide not to purchase the innovation. In this case, by substituting \( c_1 = c - \varepsilon \) and \( c_{j,1} = c \), where \( c_{j,1} \) denotes the cost of any firm excluding firm 1, into equations (1) and (2) we produce the new Cournot equilibrium quantities (superscripted NL for no licensing)

\[
(3a) \quad q_1^{NL} = \frac{a-c+\varepsilon}{(J+1)} \quad \text{and} \quad (3b) \quad q_j^{NL} = \frac{a-c-\varepsilon}{(J+1)}.
\]

The corresponding equilibrium profit functions are

\[
(4a) \quad \pi_1^{NL} = \left[ \frac{a-c+\varepsilon}{(J+1)} \right]^2 \quad \text{and} \quad (4b) \quad \pi_j^{NL} = \left[ \frac{a-c-\varepsilon}{(J+1)} \right]^2.
\]

If we next consider the case in which firm 1 licenses the innovation to all the firms in the industry in exchange for a fixed fee from each company, we can find the maximum fee that a firm would be willing to pay. Under this scenario \( c_j = c - \varepsilon \). By substituting this expression into (1) and (2) we can again find the equilibrium quantity and profit for each firm, denoted with superscript F for fee licensing.

\[
(5) \quad q_j^F = \frac{a-c+\varepsilon}{(J+1)} \quad \text{and} \quad (6) \quad \pi_j^F = \left[ \frac{a-c+\varepsilon}{(J+1)} \right]^2.
\]
By subtracting $\pi_{j,1}^{NL}$ from $\pi_{j}^{F}$, we can compute the maximum fee, $F$, that firm 1 is able to charge where firm $j$ is indifferent between purchasing or not purchasing the innovation.

$$\begin{equation}
(7) \quad F = \left[ \frac{a-c+\varepsilon}{(j+1)} \right]^2 - \left[ \frac{a-c-\varepsilon}{(j+1)} \right]^2 = \frac{4(a-c)\varepsilon}{(j+1)^2}.
\end{equation}$$

Knowing the maximum fee that firm 1 can charge each of the other firms in the industry, we can now calculate firm 1’s total income under the fixed fee licensing scheme as

$$\begin{equation}
(8) \quad \pi_{1}^{F} + (j-1)F = \left[ \frac{a-c+\varepsilon}{(j+1)} \right]^2 + (j-1)\left[ \frac{4(a-c)\varepsilon}{(j+1)^2} \right].
\end{equation}$$

We now look at the scenario in which firm 1 charges the other firms a per unit royalty on the innovation. The amount of the royalty, $r$, will be constrained by the expression $0 \leq r \leq \varepsilon$. The unit cost for firm 1 remains $c - \varepsilon$ while for all other firms the cost function becomes $c - \varepsilon + r$.

Assuming that all firms in the industry agree to implement the royalty licensing, we can again determine the equilibrium quantity and profit for each firm, superscripted by $R$ for royalty, by substituting the above cost functions into eqs. (1) and (2).

$$\begin{align}
\text{(9a)} \quad q_{1}^{R} &= \frac{a-c+\varepsilon+r(j-1)}{(j+1)} \\
\text{(9b)} \quad q_{j,1}^{R} &= \frac{a-c+\varepsilon-2r}{(j+1)} \\
\text{(10a)} \quad \pi_{1}^{R} &= \left[ \frac{a-c+\varepsilon+r(j-1)}{(j+1)} \right]^2 \\
\text{(10b)} \quad \pi_{j,1}^{R} &= \left[ \frac{a-c+\varepsilon-2r}{(j+1)} \right]^2.
\end{align}$$

We are now ready to calculate firm 1’s total income under the royalty system.

$$\begin{equation}
(11) \quad \pi_{1}^{R} + (j-1)r q_{j,1}^{R} = \left[ \frac{a-c+\varepsilon+r(j-1)}{(j+1)} \right]^2 + (j-1)r \frac{a-c+\varepsilon-2r}{(j+1)}.
\end{equation}$$

In order to maximize income, firm 1 sets $r = \varepsilon$, which gives us the expression for firm 1’s total income as
(12) \( \pi_1^R + (J - 1)r_q^R = \left[ \frac{a-c+j\epsilon}{(J+1)} \right]^2 + (J - 1)\epsilon \frac{a-c-\epsilon}{(J+1)} \).

4. Comparing fee and royalty licensing

Now that we have formulated the total income for firm 1 under both types of licensing we can compare the two to see which produces a superior outcome for the innovating firm. To do so we take the difference between the total income firm 1 receives in each scenario.

\[(13) \quad (\pi_i^R + (J - 1)r_q^R) - (\pi_i^F + (J - 1)F) = \left[ \frac{a-c+j\epsilon}{(J+1)} \right]^2 + (J - 1)\epsilon \frac{a-c-\epsilon}{(J+1)} - \left[ \frac{a-c+j\epsilon}{(J+1)} \right]^2 + (J - 1)\epsilon \frac{4(a-c-\epsilon)}{(J+1)^2} \]

\[= \frac{\epsilon(a-c)(J-1)^2}{(J+1)^2} \]

This result shows us that royalty licensing is superior to fixed fee licensing for firm 1 even when the model is extended to include an arbitrary number of firms. Looking further we can see that as \( J \) approaches infinity, the ratio of the squared terms in the numerator and denominator will approach 1. Therefore, as long as \( a > c \), royalty licensing will be more beneficial than fixed fee licensing. This result differs from that of Wang, who states that fixed fee licensing will eventually become preferable as the number of firms in the industry becomes large. Given this result I conclude that the superiority of royalty licensing over fixed fee licensing survives the extension to an arbitrary number of firms, and will maintain this advantage even as the number of firms approaches infinity.

References
